INSTITUT DES HAUTES ÉTUDES

POUR LE DÉVELOPPEMENT DE LA CULTURE, DE LA SCIENCE ET DE LA TECHNOLOGIE EN BULGARIE

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Concours Général de Physiques
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The two problems are completely independent and can be treated in any order.
The clearness, the quality and the precision of the work, which must be written in ***English***, will form a ***very significant*** share of the final note.
The duration of the composition is ***4 hours***.

Problem 1: The human sight.

The human eye is composed in a first approximation of a lens and a detector (retina) as shown on *figure 1*. The size of the human eye is approximately D=24mm of diameter.

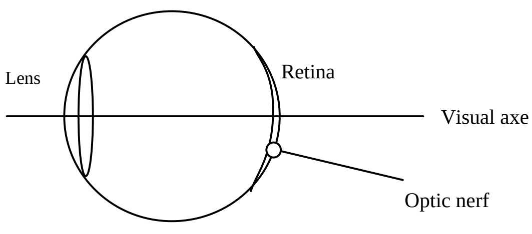

Figure 1

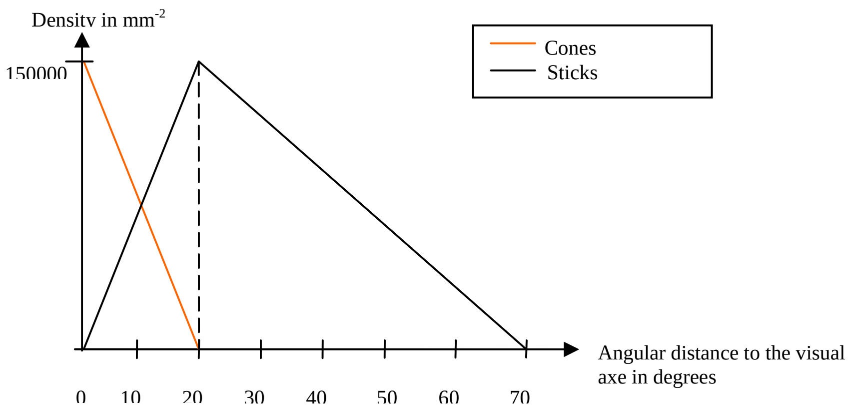

Figure 2

The detector is made of two types of small cells called cones and sticks. The cones are present especially in the centre of the visual axis while the sticks are present in a broader zone and their concentration is maximum at about 20° optical axis. *Figure 2* shows the distribution of the cones and the sticks on the retina.

First part : In this part only the cones are considered. It is also supposed that the retina is plane and that it is located at a distance D behind the lens.

1. What must be the focal power of the lens so that an object located ad infinitum is focused on the retina.
2. An object is placed at a distance of 1m. from the eye.
   a. What must be the focal distance of the lens so that the object is clearly seen?

b. What is the minimal size of the object so that it could be clearly seen?

3. The retina of a short-sighted person focuses an object located at 3m of the eye 1 mm in front of the retina.

   a. This defect is corrected by wearing eye-glasses. If the distance between the eye and the glasses is 1 cm, what must be the focal distance of the lens of the glasses?

   b. If contact lenses are worn, what must be the focal distance of the lens of contacts?

4. The density of the cones is supposed equal to the average density of the cones in the zone of clear vision.

   a. What is the average size of one cone?

   b. What is the minimal visible size of an object?

   c. What is the minimal distance between two points located at 1m of a normal eye so that they would be seen separately?

   d. What is the angular resolution of a healthy eye?

Second part : In this part the retina is no longer approximated to a plane and a spherical retina is considered. Moreover one takes into account the response of the cones and the sticks. It is pointed out that the surface of a portion of sphere is given by the formula $S = \Omega R^2$ with $\Omega = 2\pi(1-\cos\theta)$ where Ω is the solid angle and θ is the angle shown on *figure 3*. Furthermore, it is supposed that during daytime two sticks are necessary to have the same sensitivity as one cone.

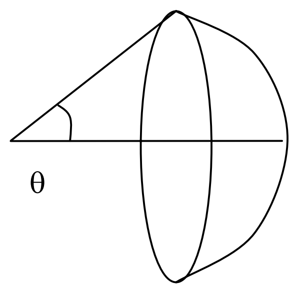

Figure 3

1. Give the angular resolution of the eye in the zone of clear vision in function of the angle of incidence of the light.
2. Calculate the angular resolution of the eye on the entire retina as a function of the angle of incidence of the light. What happens when an object moves away from the visual axis?

Problem 2 : Blood circulation

In this part of the problem, one is interested in blood circulation in the arteries. An artery is modelled by a pipe of radius r and length, with a flow Q of blood. When the blood passes through pipe, the fluid is exposed to a force of viscous friction on against the wall. This force is responsible for a fall of pressure ΔP between the two ends of the artery. We can thus define a viscous resistivity R^h.

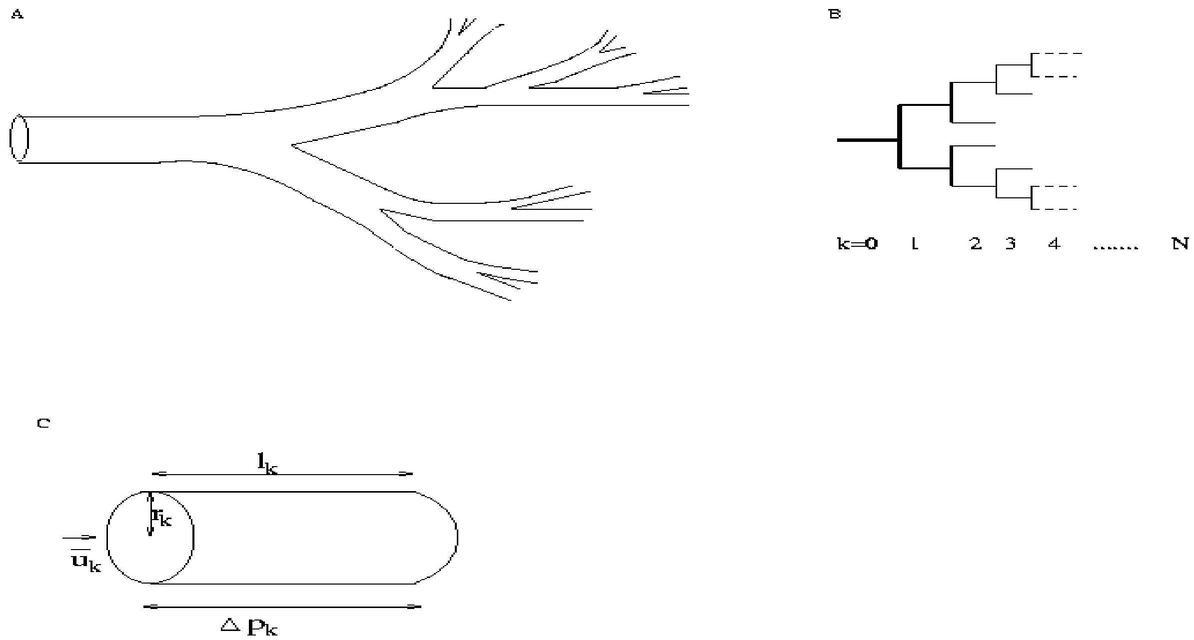

Figure 4. ***A.*** *A schematic representation of the circulatory system of a human being,* ***B.*** *representation of the tree, k specifies the order of connection in the tree the aorta being in the beginning (k=0) and the capillaries being at the end (k=N),* ***C.*** *Typical parameters a k – order vessel.*

It can be shown that there is an equivalence between this hydrodynamic circuit and an electric circuit with the proviso of associating a current I to the flow Q, a tension U to the fall of pressure ΔP and a resistance R to the hydrodynamic resistance R^h.

1. Write down Ohm's law for one artery.
2. Determine the power provided by the heart. Take Q = 5 l/min and ΔP = 100 mmHg.

A network of arteries is considered: each artery of the order k is divided into n_k arteries of order k+1 and of diameter smaller than that of the artery from which they result. The aorta will indicate the artery of order *0*; the smallest ramifications i.e. the last generation of arteries will be indicated as capillaries. A typical artery of a level of division k has a length l_k, a radius r_k and a fall of pressure Δp_k *(figure 4)*. The flow in an artery is defined as the product of the surface of a section of the artery and the mean velocity of the fluid in that section, i.e. $Q_k = \pi r_k^2 \overline{u}_k$.

3. Determine the total number N_k of arteries in the generation k?
4. Schematically, write down the law of conservation of flow?

We shall consider the scale factors $\beta_k = r_{k+1}/r_k$ and $\gamma_k = l_{k+1}/l_k$. We shall suppose that these scale factors do not depend on k, that is to say: $\beta_k = \beta$, $\gamma_k = \gamma$ et $n_k = n$.

5. How does β vary with respect to n?

By considerations which won't be detailed here, one can determine the variation of γ in function of n, $\gamma = n^{-1/3}$.

We will try in the following questions to determine by another method the way in which β and γ vary with n.

6. Write down the total volume of the network (the volume of the blood) V_b as a function of n, β, γ, N et V_0. N is the total number of generations and V_0 is the volume of the aorta. Simplify this relation when $n\gamma\beta^2 < 1$ and $N >> 1$. It is the simplified relation which will be used from now on.

The flow in the system is supposed laminar, and in this case the hydrodynamic resistance of an artery is given by Poiseuille's law $R^h{}_k = 8\mu l_k / \pi r_k^4$, where μ is the dynamic viscosity of the fluid.

7. Determine the resistance of the whole network according to n, β, γ, N and the total resistance of the aorta $R^h{}_0$. Simplify this relation when $\gamma/n\beta^4 < 1$, et $N >> 1$.
8. By making an analogy with Ohm's law determine cardiac expenditure Wc, which is, the power provided by the heart to overcome the viscous forces.

If the circulatory system is examined, keeping in mind the idea of economy, one is convinced easily that there are two antagonistic principal factors: if the artery is too small, the work necessary to make the blood circulate will be too significant (as described by Poiseuille's law), on the other hand if the volume of the vessel is too large, the volume of the blood which is stored in it will be a burden to the entire body. Determine thus the most advantageous situation in order to minimize the funcrion $F(n) = W_c + b.V_b$, where b is a dimensional constant.

9. Determine the variation of β and γ according to n. Compare those to the resulting ones previously obtained.

Now we shall consider now the distribution of the pressure along the arterial tree.

It was seen previously that when considering all the arteries of a given generation, one can replace them by an equivalent artery. As a consequence, the whole arterial network will be equivalent to a circuit of arteries in series. In what follows all the arteries shall be taken with the same thickness D and the same Young modulus E.

Recall: The Young modulus is defined as a pressure and it reflects the elastic properties of material.

10. Find a relation between the average pressure in an artery – P, E, d and the radius r of the artery. The pressure outside the vessel is considered equal to zero.
11. By using Poiseuille's law, determine the variation (decrease) of the average pressure in the arterial tree with respect to the length.
12. This law gives a rather good approximation for the pressure with respect to the length, but it does not give a correct relation between the pressure and flow at all. Do you have an idea why?

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